What is Bayes' Theorem?

Worksheet

Bayes' theorem states $P(A|B) = P(B|A)P(A) / P(B)$ - it lets you calculate the probability of A given B by reversing a known conditional probability.

P(A|B) = @frac{P(B|A) @cdot P(A)}{P(B)}

Questions

1. Bayes' theorem is used to calculate

A) a fixed probability
B) an updated (posterior) probability from new evidence
C) the mean of a dataset
D) a standard deviation

2. In $P(A|B) = P(B|A)P(A)/P(B)$, what does $P(B)$ represent?

A) The prior
B) The likelihood
C) The marginal probability of the evidence
D) The posterior

3. A test is 99% accurate but the disease is rare (0.1% prevalence). A positive result is

A) almost certainly correct
B) often a false positive due to the low base rate
C) irrelevant
D) always negative

4. If $P(B|A) = 0.8$, $P(A) = 0.25$, $P(B) = 0.4$, what is $P(A|B)$?

A) 0.5
B) 0.32
C) 0.2
D) 0.8

5. A disease affects 1% of a population. A test is 90% sensitive and has a 5% false-positive rate. If a person tests positive, what is the probability they have the disease?

6. Machine A makes 60% of products with a 2% defect rate; Machine B makes 40% with a 5% defect rate. A random product is defective - what is the probability it came from Machine B?

7. 20% of emails are spam. 30% of spam emails contain the word 'free'; 5% of legitimate emails contain 'free'. An email contains 'free' - what is the probability it's spam?

8. Define: What is Bayes' theorem?

9. Define: What is a 'prior' in Bayes' theorem?

10. Define: What is a 'posterior' in Bayes' theorem?

Answer Key

1. B) an updated (posterior) probability from new evidence - It updates prior beliefs using new evidence to produce a posterior probability.

2. C) The marginal probability of the evidence - P(B) is the total (marginal) probability of observing the evidence B.

3. B) often a false positive due to the low base rate - With a low prior, even a low false-positive rate produces many false positives relative to true positives - the base rate fallacy.

4. A) 0.5 - P(A|B) = (0.80.25)/0.4 = 0.2/0.4 = 0.5.

5. P(Disease) = 0.01, P(No disease) = 0.99 P(Positive|Disease) = 0.90, P(Positive|No disease) = 0.05 P(Positive) = 0.900.01 + 0.050.99 = 0.009 + 0.0495 = 0.0585 P(Disease|Positive) = 0.009/0.0585 0.154 (about 15.4%)

6. P(A) = 0.6, P(B) = 0.4 P(Defect|A) = 0.02, P(Defect|B) = 0.05 P(Defect) = 0.60.02 + 0.40.05 = 0.012 + 0.02 = 0.032 P(B|Defect) = 0.02/0.032 = 0.625 (62.5%)

7. P(Spam) = 0.2, P(Legit) = 0.8 P(free|Spam) = 0.3, P(free|Legit) = 0.05 P(free) = 0.30.2 + 0.050.8 = 0.06 + 0.04 = 0.10 P(Spam|free) = 0.06/0.10 = 0.6 (60%)

8. A formula that updates the probability of a hypothesis given new evidence: P(A|B) = P(B|A)P(A)/P(B).

9. P(A), the initial probability before seeing new evidence.

10. P(A|B), the updated probability after accounting for evidence B.