What is a Confidence Interval?

Worksheet

A confidence interval is calculated as CI = x z(/n), giving a range that is likely, at the chosen confidence level, to contain the true population mean.

$$\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

Questions

1. A 95% confidence interval means

A) 95% of the data falls in the interval
B) about 95% of such intervals would contain the true parameter
C) the sample mean is 95% accurate
D) there's a 95% chance the interval is wrong

2. Increasing the sample size n

A) widens the confidence interval
B) narrows the confidence interval
C) has no effect
D) only affects the mean

3. The z-score for a 99% confidence level is approximately

A) 1.645
B) 1.96
C) 2.576
D) 3.00

4. Margin of error E is calculated as

A) zn
B) z(/n)
C) z/n
D) /z

5. A sample of n=64 students has mean test score x=78, =12. Find the 95% confidence interval (z=1.96).

6. A factory samples n=100 bolts with mean length x=50.2 mm, =1.5 mm. Find the 90% CI (z=1.645).

7. A poll of n=400 voters finds x=52% support, =25%. Find the 99% CI (z=2.576).

8. Define: What is a confidence interval?

9. Define: What does a 95% confidence level mean?

10. Define: What is the margin of error?

Answer Key

1. B) about 95% of such intervals would contain the true parameter - It's about the long-run capture rate of the method, not a single interval's probability.

2. B) narrows the confidence interval - Larger n reduces /n, shrinking the margin of error.

3. C) 2.576 - z 2.576 corresponds to 99% confidence (two-tailed).

4. B) z(/n) - E = z(/n), the z-score times the standard error.

5. E = z(/n) = 1.96(12/64) = 1.96(12/8) = 1.961.5 = 2.94 CI = 78 2.94 = (75.06, 80.94)

6. E = 1.645(1.5/100) = 1.645(1.5/10) = 1.6450.15 = 0.247 CI = 50.2 0.247 = (49.953, 50.447)

7. E = 2.576(25/400) = 2.576(25/20) = 2.5761.25 = 3.22 CI = 52% 3.22% = (48.78%, 55.22%)

8. A range of values, built from sample data, likely to contain the true population parameter.

9. If you repeated the sampling many times, about 95% of the resulting intervals would contain the true parameter.

10. The value added to and subtracted from the sample mean: E = z(/n).